MIREX2007 - GRAPH SPECTRAL METHOD

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ABSTRACT

We present a graph spectral approach in which melodies are represented as graphs, based on the intervals between the notes they are composed of. These graphs are then indexed into a database using their laplacian spectra as a feature vector. This laplacian spectrum is a good representative of the original melody. Consequently, range searching around the query spectrum returns similar melodies.

1 REPRESENTATION

Our goal is to provide a sufficiently abstract representation of a melodic line that actually makes sense from a musical point of view. With this aim, we start looking just at melodies, not considering the rhythm. Melodies are generally studied from a pitch sequence/contour point of view. Our approach is different: we take as a starting point the interval structure, by which we mean the network of connections between pitches. We remark that melodies use only a subset of all possible connections, and with different frequencies. To model such relationships we use graphs, which have various and significant applications throughout mathematics, computer science, and physics. As such, the graph is a projection of the time-dependent concept of melody to a time-independent concept of intervallic structure. The next level of abstraction is to leave out pitch class information so that only the "interval connectivity" of the melody remains, and this means that certain operations such as inversion, transposition, retrogradation, other kind of permutations in the pitch class set and (some) shifting of fragments does not affect the graph. In this perspective what we are modelling is a global, time-independent signature of the melody [8], [6] [9]. Melodies that display a similar interval behaviour have similar graphs, for example melodies in which there are one or two central notes (with many connections) and a number of peripheral notes (few connections).

Let M be a melodic sequence of length $m = |M|$ and consider the sequence of pitches $\{p_j\}_{j\in I}$, $\{I = 1, ..., m\}$. Then let $V = \mathbb{Z}_{12}$ be the (metric) space of pitches, or pitch classes, in the 12-tone system. We define the graph G with vertex set $V_G = V$ and edge set whose elements are the edges a_j such that

$$a_j : \begin{cases} p_j \to p_{j+1} & \text{for every couple } (p_j, p_{j+1}) \subseteq M \\ p_m \to p_1 & \text{for the couple } (p_m, p_1) \end{cases}$$

where $j = 1, \ldots, m-1$ (see also [1] and [4]).

The arrow $a_m : p_m \to p_1$ does not represent an actual interval in the melody but it has been added for symmetry reasons and in order to take into account the relationship between the last and the first note as well, which otherwise would not have been reflected in the model.

2 INDEXING

The graph representation described up to now is a geometric one. In order to allow computations with this representation, we need to associate an algebraic structure to it. The most common algebraic structure to represent a graph is the adjacency matrix.

The adjacency matrix $A(G)$ of a graph G is a square matrix of size equal to the order of the graph and where the entry (i, j) represents the number of oriented edges from vertex i to vertex j. This adjacency matrix therefore contains all the information to reconstruct the connectivity of the graph. A matrix closely related to the adjacency matrix is the laplacian matrix $L(G)$, computed as $L(G) = D(G) - A(G)$, where $D(G)$ is the degree matrix of G. The degree matrix is also a square matrix of size equal to the order of the graph, but all values are zero except for those on the main diagonal. Here, the entry (i, i) represents the number of outgoing edges of vertex i.

Given the laplacian matrix of a melody graph, the question remains how to compute the similarity to another melody. For this purpose, we first compute the eigenvalues of the laplacian matrix and sort them by magnitude. [1] Hereby, we obtain the laplacian spectrum of the graph, that is known to reflect a number of important properties of the graph. These properties include the diameter (related to the second smallest eigenvalue), mean distance, minimum degree and algebraic connectivity. Furthermore, the spectrum is invariant under permutations of the matrix (i.e. swapping columns or rows). Together with the absence of pitch information stored in the matrix, this makes the representation invariant under transpositions and note permutation. This is an important property,

[1] Since the graphs are directed, the laplacian matrix is not necessarily symmetric. Consequently, some of the eigenvalues may be complex numbers and there exist multiple strategies for sorting these. As in [10], we sort these eigenvalues by modulus.

because as pointed out before, our concept of similarity is also independent from note permutation and transposition.
Our main motivation for encoding the topology of a graph using the laplacian matrix comes from the fact that laplacian matrices are more natural, more important, and more informative than other matrices about the input graphs [7]. Previously, Godsil and McKay [3] and more recently Haemers and Spence [5] have also shown that the laplacian matrix has more representational power than the adjacency matrix, in terms of resulting in fewer cospectral graphs. Recall that two graphs are called cospectral (or, isospectral) if they have the same eigenvalues.

Given a query graph and a large database, the objective of an indexing algorithm is to efficiently retrieve a small set of candidate matches, that share topological similarity with the query. As pointed out, we encode the topology of a graph through its laplacian spectrum, which is used as a signature for the database object. This spectrum can be seen as a point in a high dimensional space. To compute similarity between two graphs, we compute the Euclidean distance between their signatures, which is inversely proportional to the structural similarity of the graphs. Therefore, for a given query, retrieving its similar graphs can be reduced to a nearest neighbor search among a set of points. A set of candidate matches can now be found without having to inspect the entire database. For more details on this indexing strategy, the reader is referred to [2].

3 REFERENCES

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